Physics 200 Name KEY

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Exam 3 – Rotation and Gravity

November 24, 2008

This is a closed book examination. You may use a small 3x5 card with equations on it. There is extra scratch paper available. Explanations must be included with all answers – even multiple-choice questions. Your explanation is worth 75% of the possible points.

A general reminder about problem solving:

1. Draw a picture then create a simplified free body diagram with all forces
2. Write down what you know including coordinate frame
3. Write down what you don't know and/or want to know
4. List mathematical relationships
5. Simplify and solve
6. Check your answer – Is it reasonable? Are units correct?

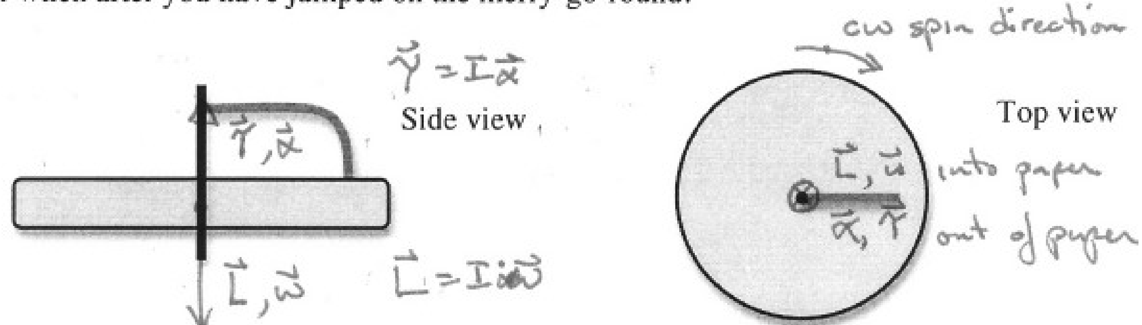
- Show all work! Use extra paper if needed.

1. [4 PTS] Given a ring and a hollow sphere that have the same mass, can they have the same moment of inertia?

   A. No
   B. Yes

   $I = \beta m r^2$ Adjust the radius so $I_{sphere} = I_{ring}$

2. [4 PTS] You are on a playground merry-go-round (a spinning disk) that you started spinning in a clockwise direction as seen from above. There is only a small frictional torque so you will have a nice long ride. Clearly draw and label (on top and side views) the rotational vectors ($\vec{L}$, $\vec{\omega}$, $\vec{\alpha}$, $\vec{\tau}$) for when after you have jumped on the merry-go-round.



$\vec{\tau}$ and hence $\vec{\alpha}$ is working to slow down merry-go-round so they are in opposite direction to $\vec{L}$ and $\vec{\omega}$

3. [4 PTS] A large heavy object sits at x=20 cm and a small light object sits at x=40 cm. You want to increase the gravitational force by a factor of 4. What must you do?

   A. Move the light object to x= 80 cm.
   B. Move the heavy object to x= 0 cm.
   C. Add twice the mass to the light object.
   D. Add twice the mass to the heavy object.
   E. Move either of the objects to x= 30 cm.

   $F_g = G \frac{m_1 m_2}{r_{12}^2}$

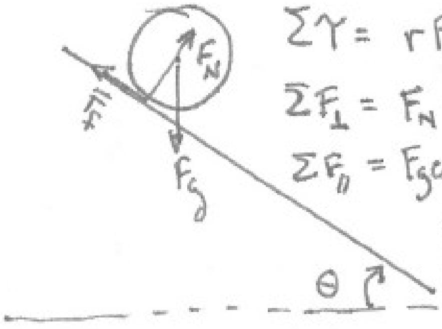
   increase F_g by decreasing separation distance r_{12}

   Initial $r_{12} = 20\text{cm}$ so since it is squared

   Final $r_{12} = (\frac{1}{2}) 20\text{cm} = 10\text{cm}$

4. [8 PTS] Two solid disks are set to roll (without slipping) down an incline. The disks are different sizes so that one disk has twice the radius of the other disk. Since both disks are made of the same material the larger disk is also more massive. When these two disks are released which one reaches the bottom of the incline first?

   A. The larger disk reaches the bottom first.
   B. Both disks reach the bottom at the same time.
   C. The smaller disk reaches the bottom first.



$\sum \tau = rF_f = I\alpha = r\mu mg\sin\theta = \beta mr^2 \frac{a}{r}$ or $F_f = (\beta mr^2)\frac{a}{r^2}$

$\sum F_\perp = F_N - F_g \sin\theta = 0$

$\sum F_\parallel = F_g\cos\theta - F_f = ma$

$F_f = \mu F_N$

$F_g = mg$

$\alpha = \frac{a}{r}$

$I = \beta mr^2$

$mg\cos\theta - \beta ma = ma$

$g\cos\theta = (1+\beta)a$

$\frac{g\cos\theta}{(1+\beta)} = a$

OR

$E_i = E_f$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$I = \beta mr^2 \quad \omega = \frac{v}{r}$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\beta mv^2$

$\left(\frac{2gh}{1+\beta}\right)^{1/2} = v$

Note $v_{roll} < v_{slide}$

velocity is

Acceleration "independent of mass and radius" (just shape)

You decide to spin a solid ball and a solid disk (see picture below). Both the disk and ball have the same radius and mass and you spin both with the same torque for 2 rotations. The next two questions involve this ball and disk. NOTE: REMEMBER TO PROVIDE EXPLANATIONS.

$I_s < I_D$

$I_s = \frac{2}{5}mr^2$

$I_D = \frac{1}{2}mr^2$

5. [4 PTS] After being spun which object has a greater angular momentum?

   A. The ball has a greater angular momentum.
   B. Both the ball and disk have the same angular momentum.
   C. The disk has a greater angular momentum.

$\int \tau \, d\theta = \text{Energy} = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{L^2}{I}$

$\therefore \frac{L_D^2}{I_D} = \frac{L_s^2}{I_s} \quad L_D = \left(\frac{I_D}{I_s}\right)^{1/2} L_s \quad L_D > L_s$

Note: $\int \tau \, dt =$ change in angular momentum $= \Delta \vec{L}$

6. [4 PTS] After being spun which object has more energy?

   A. The ball has more energy.
   B. Both the ball and disk have the same energy.
   C. The disk has more energy.

Energy $\equiv \int \tau \, d\theta$ is same for both

7. [4 PTS] You have built a device capable of launching an object (m=1 g) at a velocity (v) so that the object can escape the earth's gravity. You want to launch a larger object (m=4 g). At what velocity must you launch it?
   A. Launch at 16v
   B. Launch at 4v
   C. Launch at 2v
   D. Launch at v

$E_i = E_f$ $\quad v_i = 0$ and $r_f = \infty$ (escaped earth's gravity)

$\frac{G M_E m_2}{R_E} = \frac{1}{2} m_2 v_{esc}^2$ $\qquad v_{esc} = \sqrt{\frac{2 G M_E}{R_E}}$

mass cancels!

independent of mass...!

Note Energy ($\frac{1}{2}mv^2$) does increase

8. [12 PTS] You are standing on an asteroid that is roughly spherical with over a 22 mile circumference (r=5700 m) and the same density as the earth (m=4.3x10^{15} kg).
   A. What would be the consequence if you were able to jump with a velocity of 11.5 m/s?
   B. What would be the orbital velocity for an object orbiting 300 m above the surface of the asteroid?

(A) $v_{esc} = 10$ m/s so you launch yourself

(B) v_{orbit}(@ r=300m) = 6.9 m/s

9. [12 PTS] A "lazy susan" is a circular rotating tray (i.e. solid disk) placed on a table to help pass food around. While at your professor's house for thanksgiving you are playing with the "lazy susan" on the table. You estimate this "lazy susan" is 10 kg and has a radius of 30 cm.
   A. You push the edge of the "lazy susan" (tangentially) with a force of 50 N for about 20 degrees. Assume there is no friction slowing the rotation and the "lazy susan" starts at rest. How fast is the "lazy susan" spinning?
   B. You place a salt shaker on the "lazy susan" half-way out from the center (r=15 cm). When you spin the "lazy susan" more than 6.75 rad/sec the salt shaker slides off the tray. What is coefficient of static friction between the tray and the salt shaker?

(A) $\omega = 4.8$ rad/sec

(B) $\mu_s = 0.7$

Useful Data:
Mass of Earth = 6×10^{24} kg
Radius of the Earth = 6.4×10^{6} m
G = 6.67×10^{-11} Nm^2/kg^2

$\boxed{8}$ Determine escape velocity from asteroid.

$$E_i = E_f$$

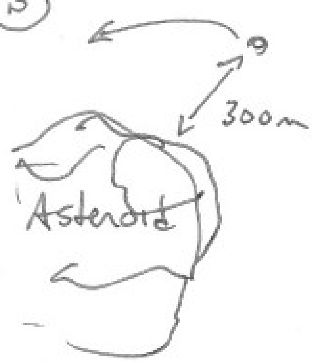
$$G\frac{M_a m}{R_a} = \frac{1}{2} m v_e^2 \qquad v_e = \left(\frac{2GM_a}{R_a}\right)^{1/2} \qquad M_a = 4.3 \times 10^{15}\,kg \quad R_a = 5700\,m$$

$\checkmark$ units

$$v_e = 10.0\ m/s$$

(A) So if you jump w/ a velocity $11.5\,m/s > V_{esc}$ you would "launch" yourself off the asteroid. oops

(3)



$$\Sigma F = ma \qquad a = \frac{v^2}{r}$$

It is in orbit only force on it is gravitational

$$\frac{GM_a m}{(R_a+r)^2} = \frac{mv^2}{(r+R_a)}$$

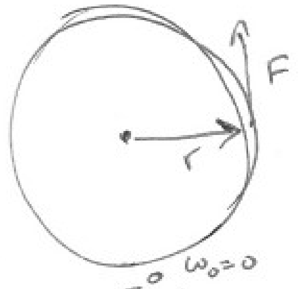
$$\left(\frac{GM_a}{R_a+r}\right)^{1/2} = v \qquad R_a + r = 6000\ m$$

$\checkmark$ units

$$v = 6.9\ m/s$$

{slower than escape velocity}

$\boxed{9}$



$$I_D = \frac{1}{2}mr^2$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\alpha = \frac{\tau}{I} = \frac{rF}{\frac{1}{2}mr^2} = \frac{2F}{mr}$$

$$\omega = \cancel{\omega_0} + \alpha t$$

$$\Delta\theta = \cancel{\omega_0 t} + \frac{1}{2}\alpha t^2 = \frac{1}{2}\alpha\left(\frac{\omega}{\alpha}\right)^2 = \frac{\omega^2}{2\alpha}$$

$$\omega = (2\alpha\Delta\theta)^{1/2} = \left(\frac{4F\Delta\theta}{mr}\right)^{1/2} \qquad \Delta\theta = 20° \cdot \frac{\pi}{180°} = \frac{\pi}{9}\ rad$$

(A)

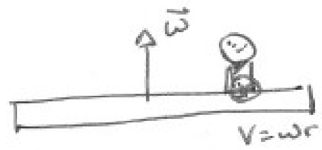
$$\omega = 4.8\ rad/sec$$

$\checkmark$ units

$\checkmark$ $F\uparrow$ $\omega\uparrow$

$m\uparrow$ $\omega\downarrow$

$\Delta\theta\uparrow$ $\omega\uparrow$



$$\mu_s F_N = F_f \qquad F_N,\ F_g$$

$$F_N = mg$$

$$\mu_s mg = \frac{mv^2}{r} = \frac{m(r\omega)^2}{r}$$

$$\cancel{m}\mu_s g = \cancel{m}\omega^2 r$$

$$\mu_s = \frac{\omega^2 r}{g}$$

$$\mu_s = 0.7$$

NOTE Your prior spin does not slide self off.

$\checkmark$ units

$\omega\uparrow$ $\mu_s\uparrow$

$r\uparrow$ $\mu_s\uparrow$